Dynamics of a sliding particle in rotating drum

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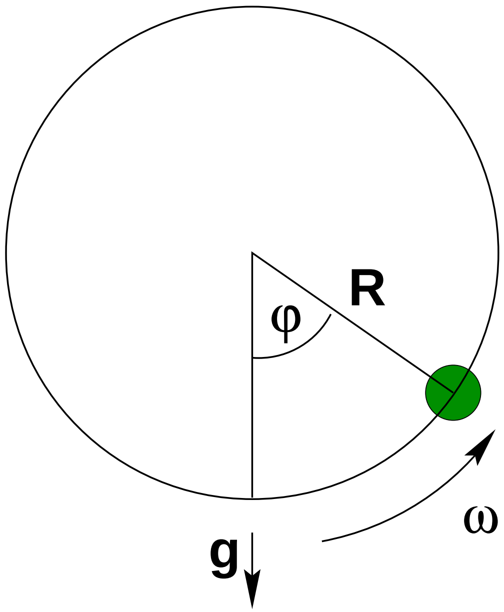
$$\ddot{\varphi}(\mathrm{t}) = -\mathrm{g}\sin\varphi(\mathrm{t}) + \mu(\dot{\varphi}_{rel}(\mathrm{t})) \cdot \left\{\mathrm{g}\cos\varphi(\mathrm{t}) + \dot{\varphi}^2(\mathrm{t})\right\}$$

Using the averaging method we

- test different friction laws
- search for periodic orbits
- investigate the structure of the phase space

At the present we are interested in better understanding the influence of friction in simulations for granular systems.
The far goal is a 3 dimensional simulation for 3 using non-spherical particles.

Introduction



We want to investigate the motion of a particle in a rotating drum and are interested in steady orbits. Also we want to test different laws for the friction force.

$$\ddot{\varphi}(\mathrm{t}) = -\mathrm{g}\sin\varphi(\mathrm{t}) + \mu(\dot{\varphi}_{rel}(\mathrm{t})) \cdot \left\{\mathrm{g}\cos\varphi(\mathrm{t}) + \dot{\varphi}^2(\mathrm{t})\right\}$$

$$= -\mathrm{g}\sin\varphi + \mu_0 \cdot \left(\mathrm{g}\cos\varphi + \dot{\varphi}^2\right) + \varepsilon \underbrace{\left\{(\mu(\dot{\varphi}) - \mu_0) \cdot \left(\mathrm{g}\cos\varphi + \dot{\varphi}^2\right)\right\}}_{perturbation}$$

The Nonperturbated System

In the first step we will integrate the nonperturbated system

$$\ddot{\varphi} = -\mathrm{g}\sin\varphi + \mu_0 \cdot \left(\mathrm{g}\cos\varphi + \dot{\varphi}^2\right)$$

We get

$$\dot{\varphi}^2 = -2\mathrm{g}\frac{1}{1+4\mu_0^2} \cdot \left\{\left(2\mu_0^2 - 1\right)\cos\varphi - 3\mu_0 \sin\varphi\right\} + 2e^{2\mu_0\varphi} \cdot \mathrm{c} \quad (1)$$

and will solve for integration constant c and calculate the first derivative

$$\dot{\mathrm{c}} = e^{-2\mu_0\varphi}\dot{\varphi}\underbrace{\left[\ddot{\varphi} + \mathrm{g}\sin\varphi - \mu_0(\mathrm{g}\cos\varphi + \dot{\varphi}^2)\right]}_{\equiv 0} \quad (2)$$

The Perturbated System

We assume that the presence of the perturbation will change the constant of integration c into a slowly varying function of time:

$$c = c_0(\tau) + \varepsilon c_1(t, \tau) + \cdots \tag{3}$$

with $\tau = \varepsilon t$ is a slow time. Differentiating once yields

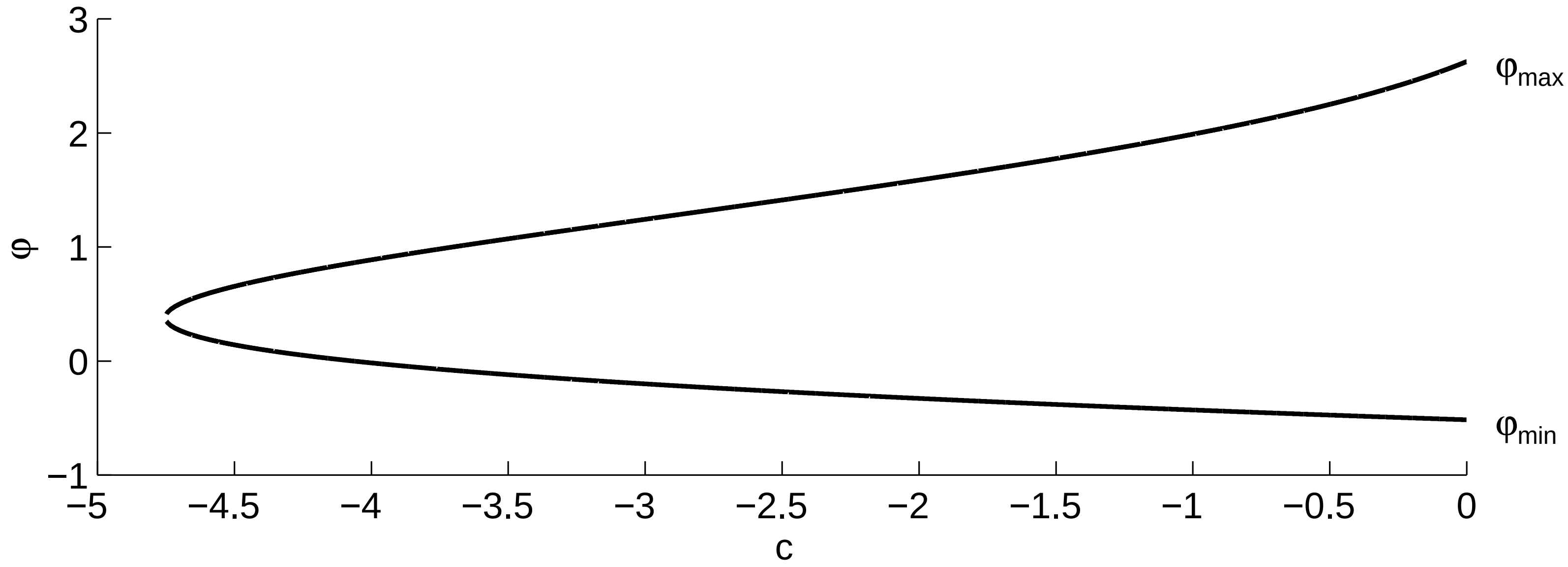
$$\dot{c} = \varepsilon(c_{0\tau}(\tau) + c_{1t}(t, \tau)) + \mathcal{O}(\varepsilon^2) \tag{4}$$

Integrating over one period, we get rid of c_1 , and using equation (2), we have

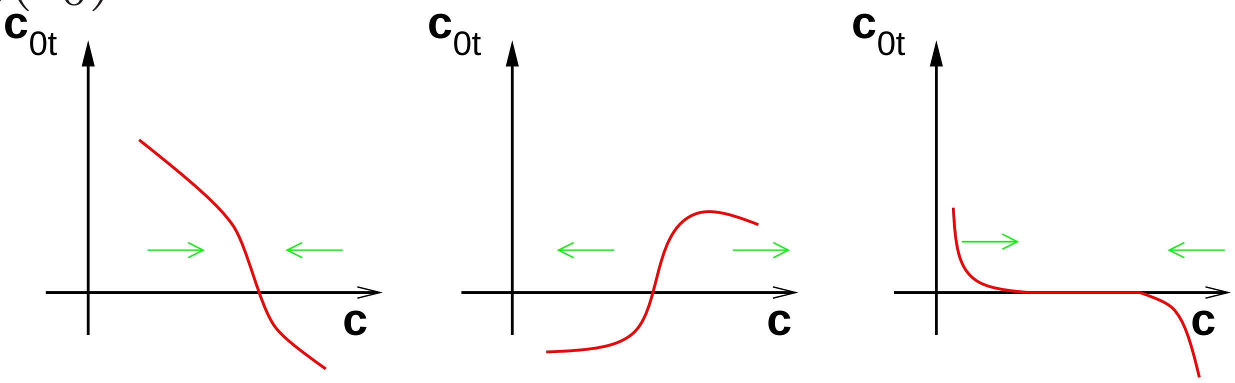
$$\varepsilon c_{0t} = -\frac{2}{\mathrm{T}_p} \int_{\varphi^m in}^{\varphi^m ax} \mathrm{d}\varphi \; \dot{\varphi} e^{-2\mu_0 \varphi} \dot{\varphi} \cdot \varepsilon \left[(\mu(\dot{\varphi}) - \mu_0) \cdot \left(\mathrm{g} \cos\varphi + \dot{\varphi}^2 \right) \right] \tag{5}$$

where T_p is the periodicity. Of course T_p itself depends on c_0 via equation (1).

$$\mathrm{T}_p = 2\int_{\varphi^{min}}^{\varphi^{max}} \mathrm{d}\varphi \, \frac{1}{\dot{\varphi}}.$$



If equation (5) happens to have a fixed point c_O^*, there is a periodic solution of the perturbated equation, the periodicity of which is given by $T_p(c_0)$.



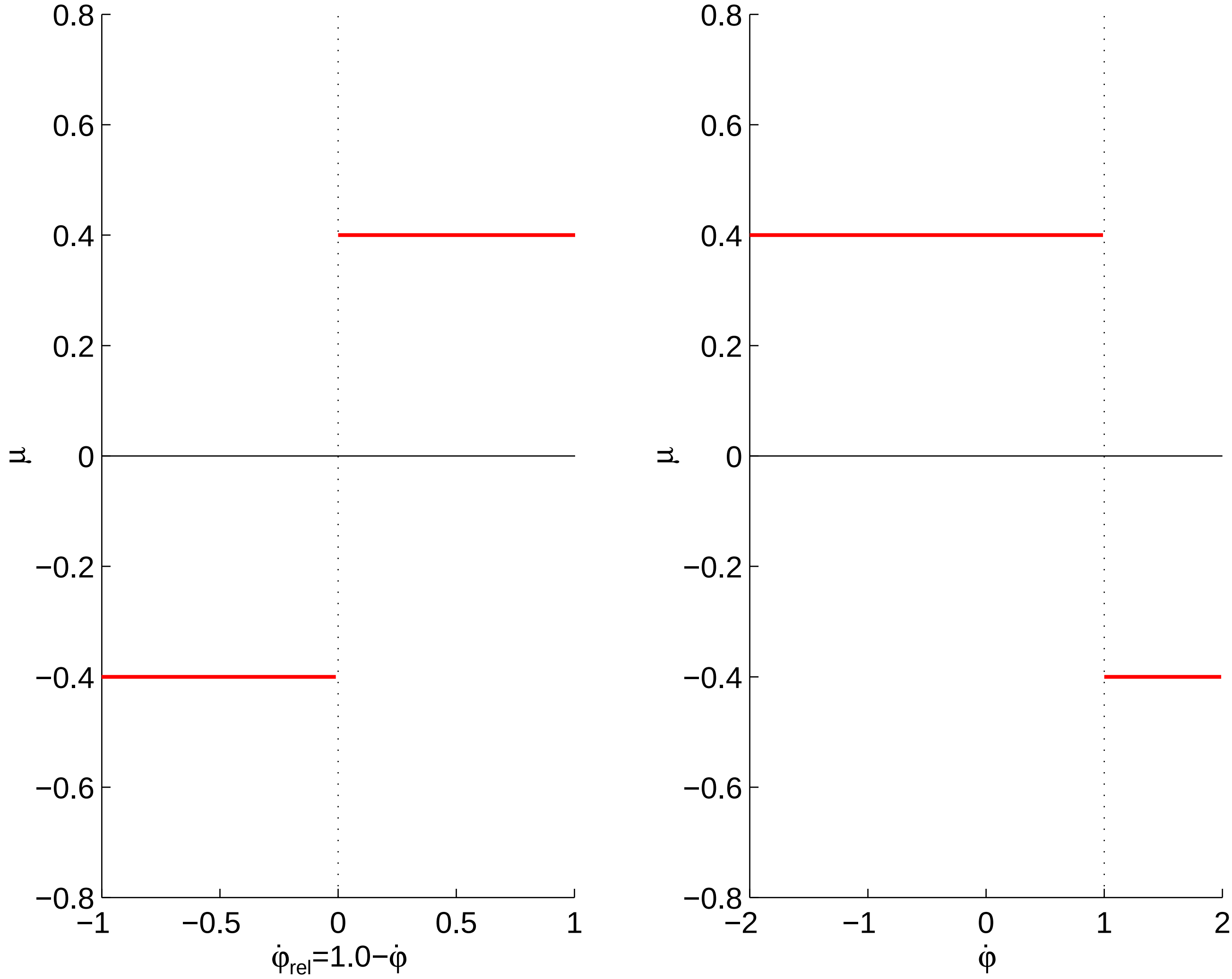
(a) (b) (c)

We have three different types of fixed points. In case (a) we have a stable orbit, case (b) is meta stable. In case (c) we have a marginally stable orbit.

Case 1

In this case the friction coefficient depends only on the direction of the velocity vector.(Coulomb's law)

$$\mu(\dot{\varphi}_{rel}) = \begin{cases} \mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} \geq 0 \\ -\mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} < 0 \end{cases} .$$



We can observe two different kinds of trajectories:

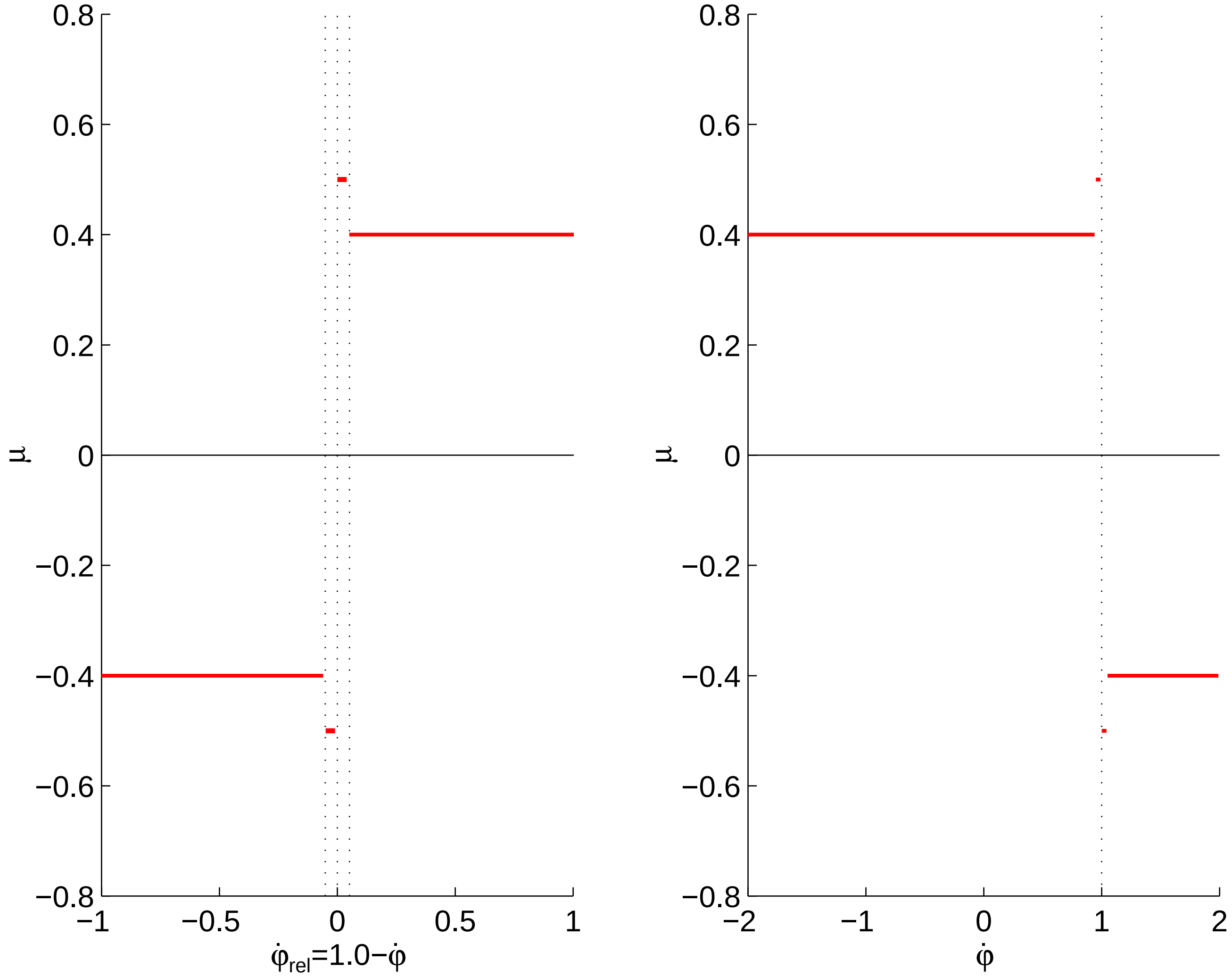
1. For small starting points $\varphi(0)$ the trajectory is marginally stable

2. outside of the periodic orbit we can dissipate energy, the trajectories are approaching the periodic orbit.

Case 2

This case is similar to case 2 but we have additionally μ_{stat} within a small range of velocity.

$$\mu(\dot{\varphi}_{rel}) = \begin{cases} \mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} \geq \mathrm{v}_0 \\ \mu_{stat} & \text{if} \quad 0 \leq \dot{\varphi}_{rel} < \mathrm{v}_0 \\ -\mu_{stat} & \text{if} \quad -\mathrm{v}_0 \leq \dot{\varphi}_{rel} < 0 \\ -\mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} < -\mathrm{v}_0 \end{cases} .$$



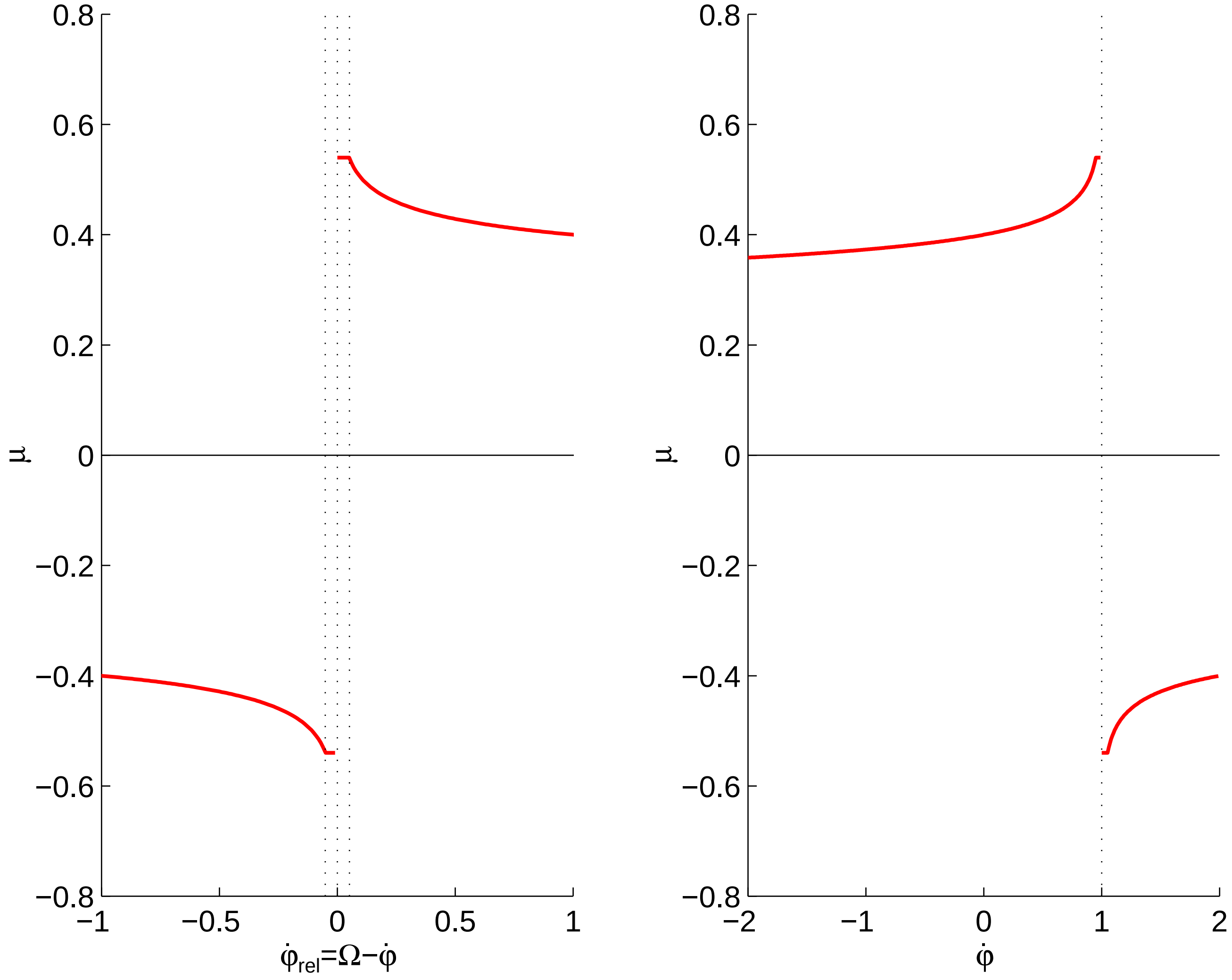
Here we have 4 different kind of trajectories:

1. for small $\varphi(0)$ the trajectory is marginally stable
2. for a small range of $\varphi(0)$ (if the trajectory can reach the area of static friction) we can gain energy and approach the stable state
3. the periodic orbit itself
4. the trajectories are approaching the stable state.

Case 3

This case is similar to case 2 but we have a velocity dependence of $\mu \propto v^{-0.1}$ (E. Rabinowicz)

$$\mu(\dot{\varphi}_{rel}) = \begin{cases} \mu_{kin} \cdot \dot{\varphi}_{rel}^{-0.1} & \text{if} \quad \dot{\varphi}_{rel} \geq \mathrm{v}_0 \\ \mu_{kin} \cdot \mathrm{v}_0^{-0.1} & \text{if} \quad 0 \leq \dot{\varphi}_{rel} < \mathrm{v}_0 \\ -\mu_{kin} \cdot \mathrm{v}_0^{-0.1} & \text{if} \quad -\mathrm{v}_0 \leq \dot{\varphi}_{rel} < 0 \\ -\mu_{kin} \cdot \dot{\varphi}_{rel}^{-0.1} & \text{if} \quad \dot{\varphi}_{rel} < -\mathrm{v}_0 \end{cases}$$



Here we have 3 different kind of trajectories:

1. for $\varphi(0) < \varphi(0)_{\text{periodic}}$ we can gain energy and approach the periodic state
2. the periodic orbit itself
3. for $\varphi(0) < \varphi(0)_{\text{periodic}}$ the trajectories are dissipating energy and approaching the stable orbit.

Conclusion

We have investigated the behavior of a particle in a rotating drum. Using a perturbativ treatment,we found differentseveral kinds of behavior for various friction laws. There are disagreements with numerical simulations made by G. Ristow. However, we found good agreement with experiments made by the Rehberg group (A. Betat).

References

[1] Nayfeh Ali (1973) *Perturbation Methods*

[2] Rabinowicz Ernest (1965) *Friction and Wear of Materials*