Rotation and Reptation

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One-particle theories and many-particle systems:

- Swirling granular matter: Reptation

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- A Sliding particle in a rotating drum: Rotation

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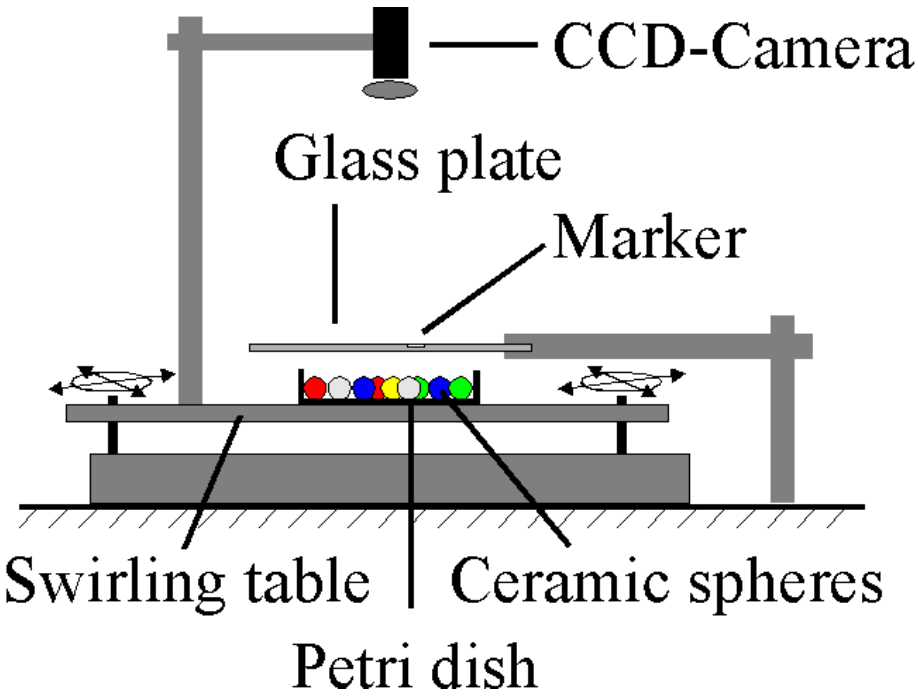
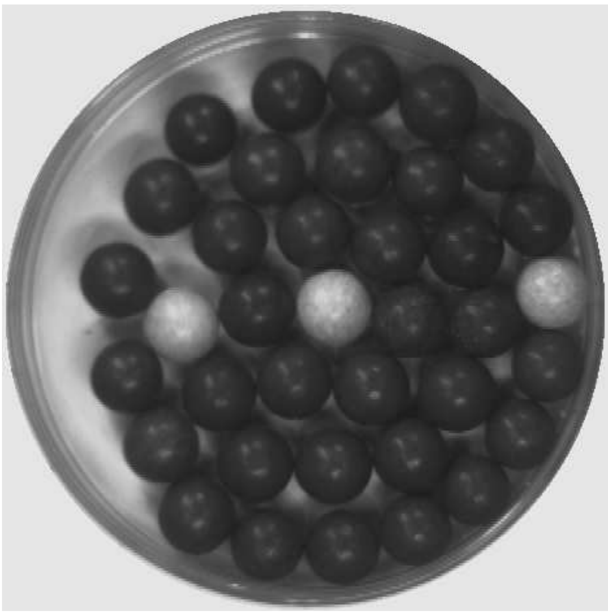
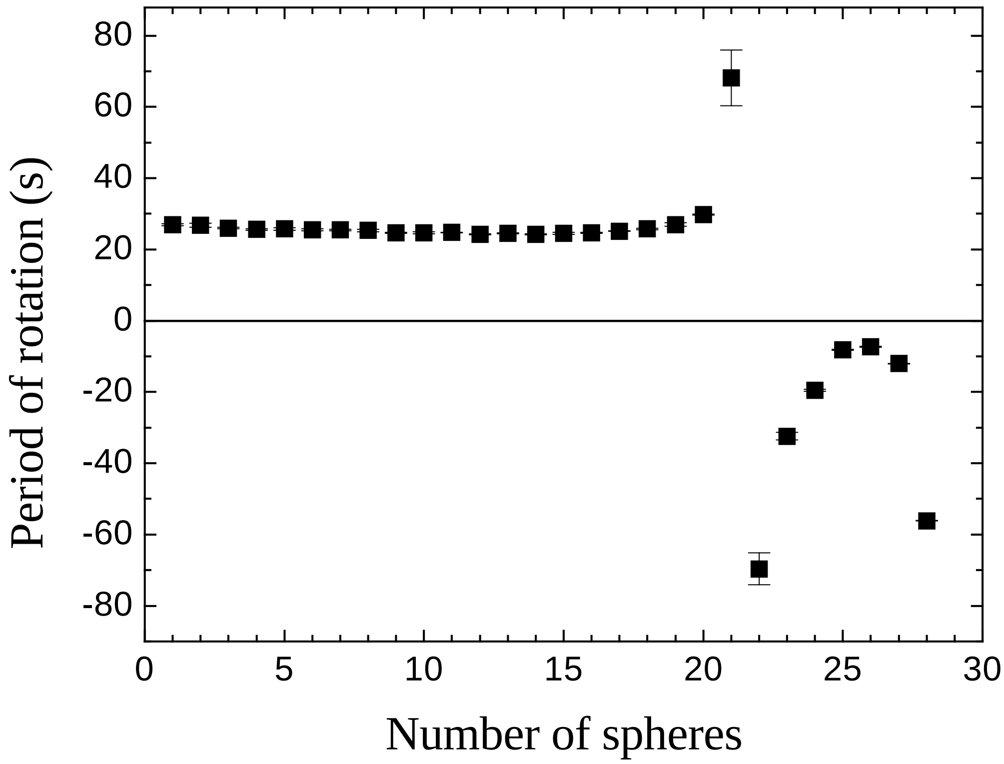
Experimental Setup

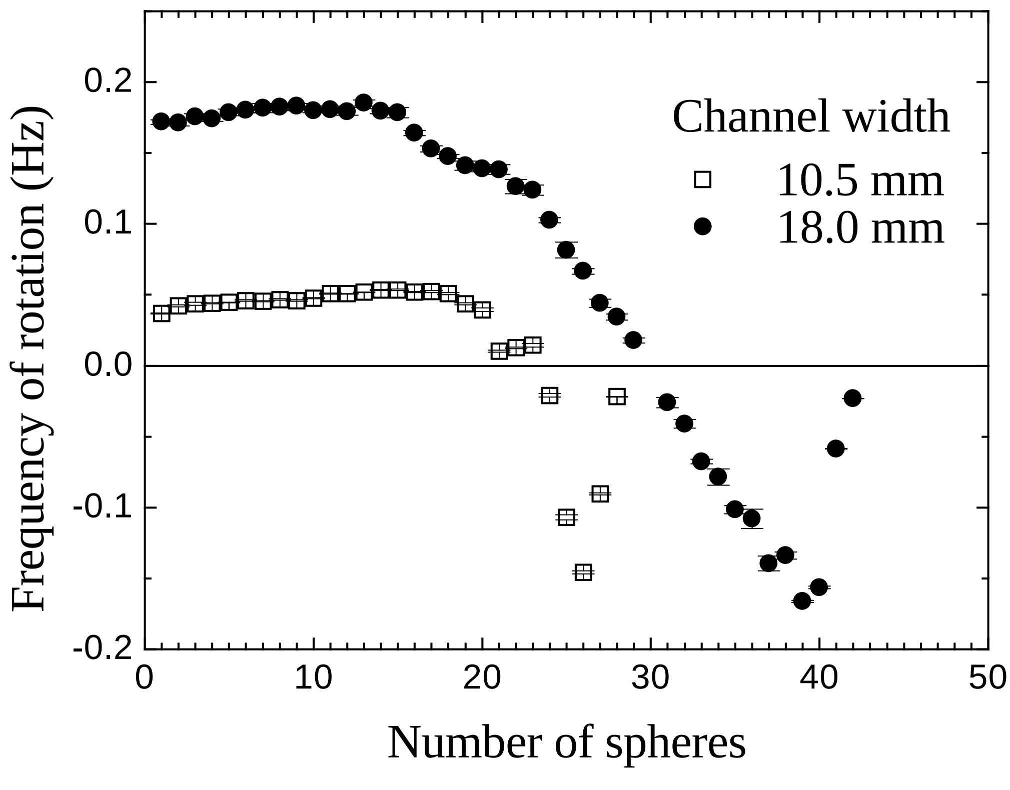


Picture seen by the CCD-camera



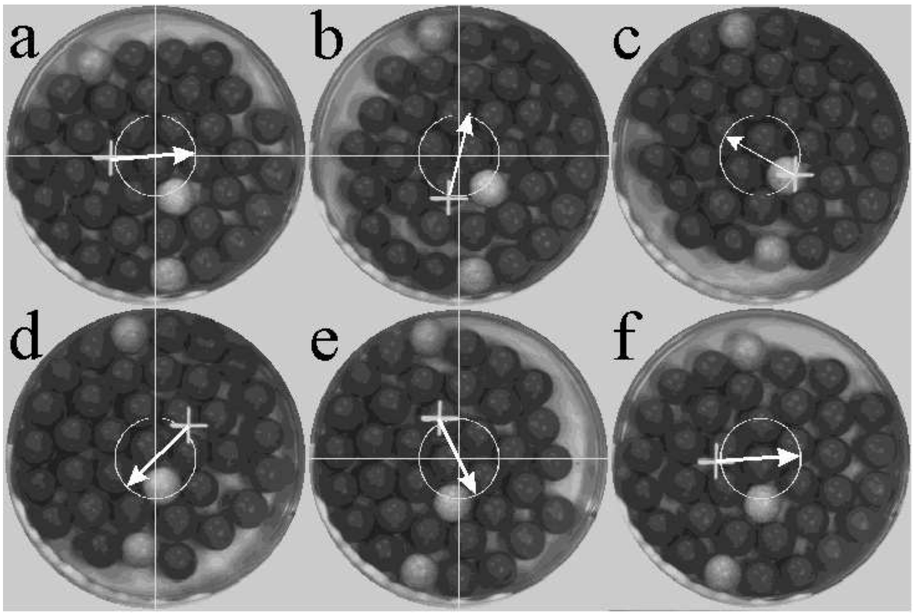


Period of rotation (s)
Number of spheres



Frequency of rotation (Hz)
Number of spheres
Channel width
10.5 mm
18.0 mm

Swirled Annulus



- reptation mode ⇒ low diffusion coefficient
- the inner cluster doesn't separate
- Can we replace it by a fixed disk?

⇒ The same phenomena is observed!

Advantages:

- The particles have a fixed relation to the neighbors.
- Better visualization.
- Center of mass can be studied on a circular line.

The reptation mode is influenced by

1. number of spheres
2. channel width
3. particle's material
4. driving frequency

But the transition rotation$\Leftrightarrow$reptation can always be found.

If you want more information on the experiment and a preprint, send an e-mail to:

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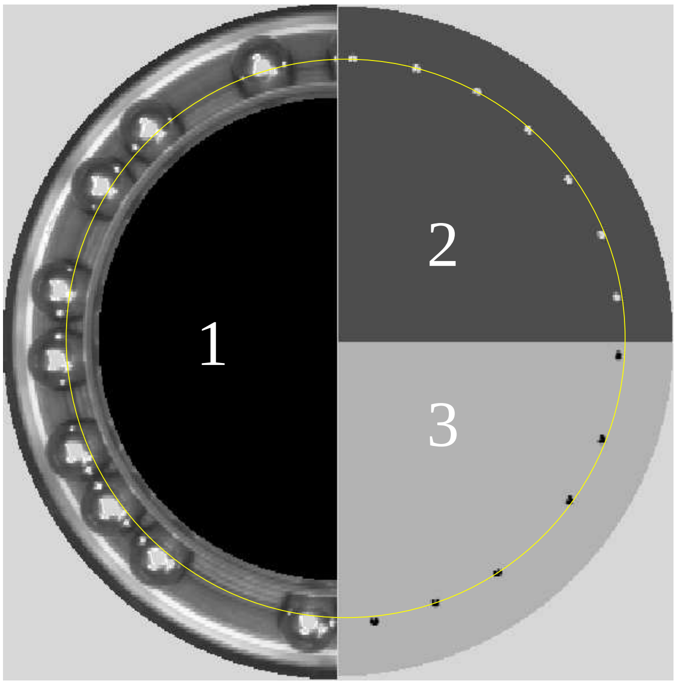
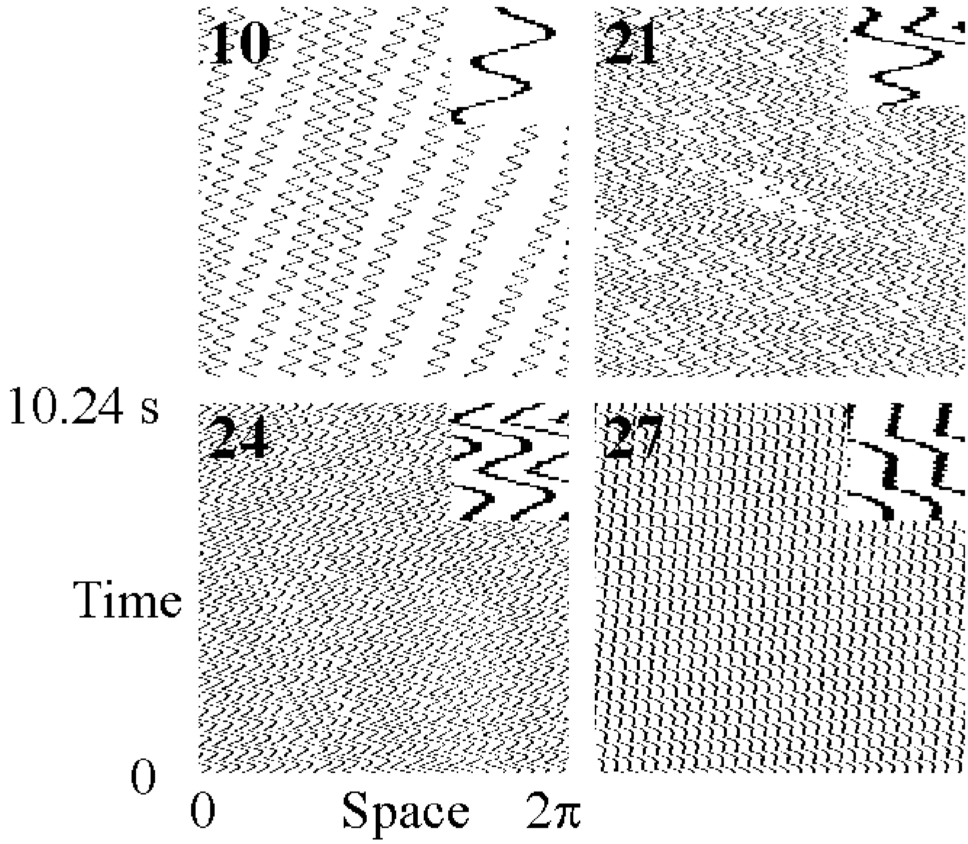
Particles Dynamics

Visualize the trajectories:



1. real image
2. real images with reduced aperture
3. inverse image of 2

Space time evolution of traces



Rotation mode

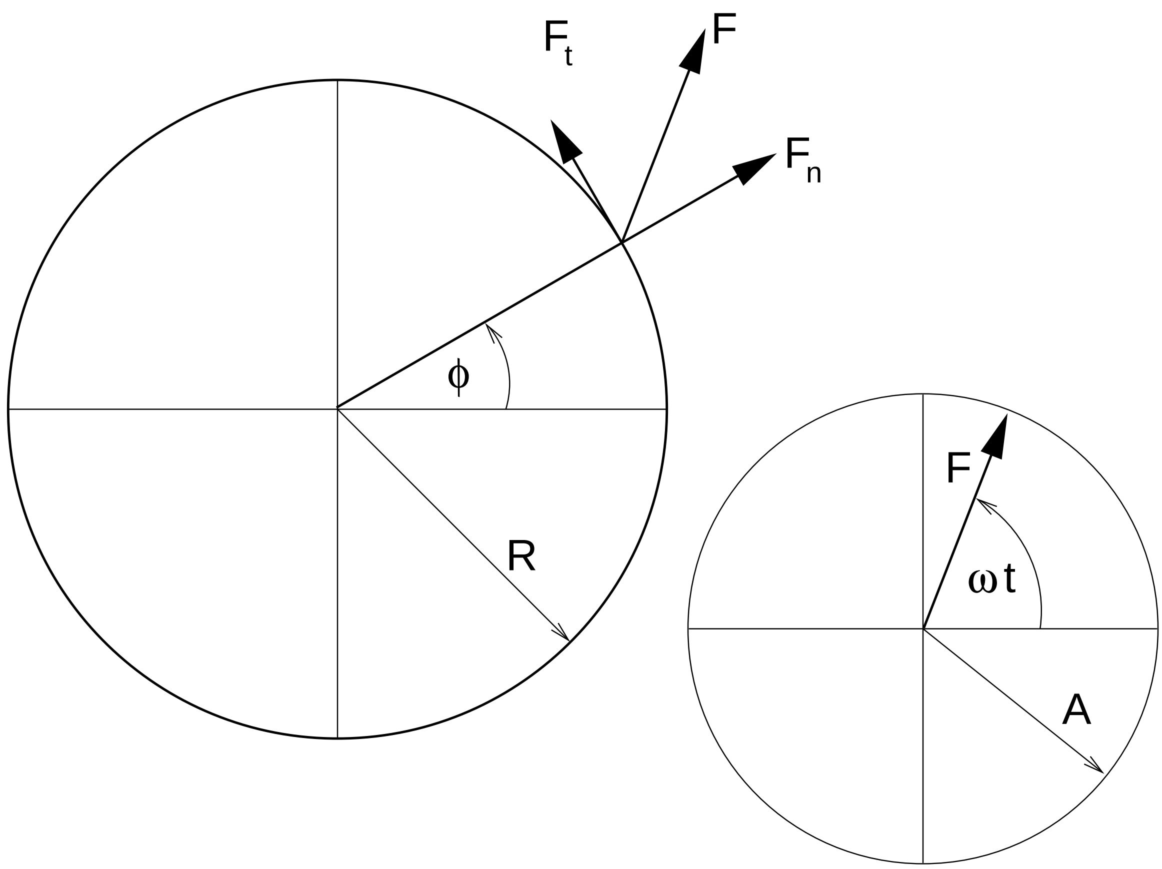
- forward motion ¿ backward motion → positive translation movement

Reptation mode

- forward motion is blocked → negative translation movement

Theory

Start with single bead in a swirled annulus:



$$m\ddot{\varphi} + \Gamma\dot{\varphi} + \frac{m\omega^2 A}{R}\sin(\varphi - \omega t)$$

Rescaling $t \to \omega t$ gives

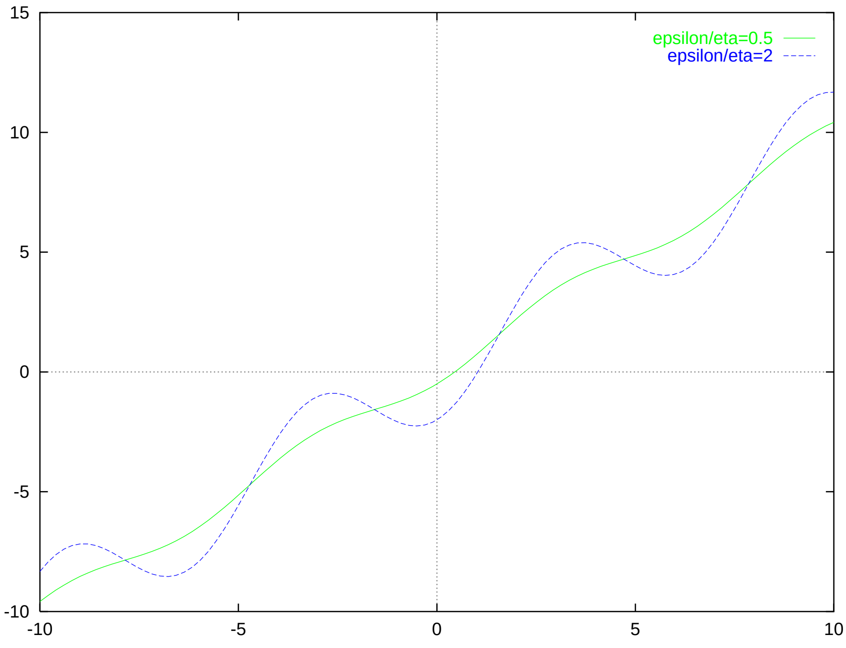
$$\ddot{\varphi} + \eta\dot{\varphi} + \varepsilon \sin(\varphi - \omega t)$$

where

$\eta = \frac{\Gamma}{m\omega}$ and $\varepsilon = \frac{A}{R}$

Two qualitatively different solutions:

- $\frac{\varepsilon}{\eta} > 1 \Rightarrow$ stationary solutions
- $\frac{\varepsilon}{\eta} < 1 \Rightarrow$ no stationary solutions, experimentally relevant



Split φ

$$\varphi(t) = \underbrace{\varphi(t)}_{\text{slow part}} + \underbrace{\delta\varphi(t)}_{\text{fast oscillation, low amplitude}}$$

We obtain

$$\ddot{\varphi} + \delta\ddot{\varphi} + \eta\dot{\varphi} + \eta\delta\dot{\varphi} \simeq -\varepsilon\sin(\varphi - t) + \varepsilon\cos(\varphi - t)\delta\varphi$$

For the fast part we find

$$\delta\ddot{\varphi} + \eta\delta\dot{\varphi} = -\varepsilon\sin(\varphi - t)$$

$$\delta\varphi = -\varepsilon\int_0^t dt'\exp(-\eta(t - t')\cos(\varphi - t))$$

Using this result and averaging over one period of the external force we arrive at

$$\ddot{\varphi} + \eta\dot{\varphi} = \frac{\varepsilon^2\eta}{2(1 + \eta^2)} - \frac{\varepsilon^2(1 - e^{-2\pi\eta})}{2\pi(1 + \eta^2)^2}((1 + \eta^2)\cos^2\varphi - 1)$$

We obtain

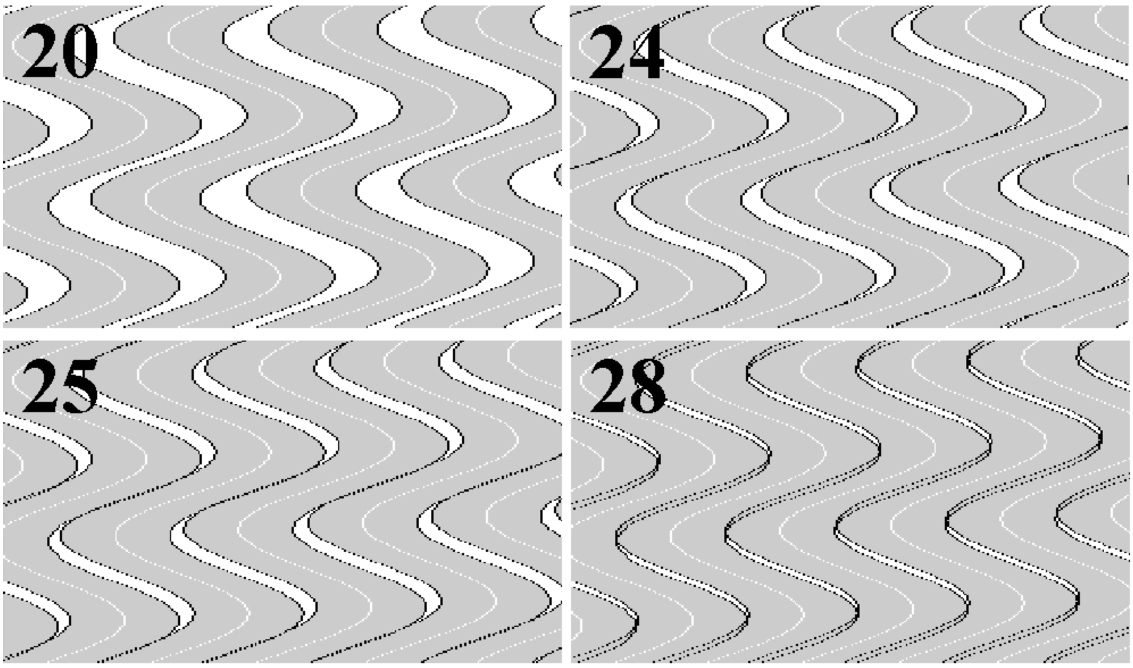
$$\varphi(t) = \frac{\varepsilon^2}{2(1+\eta^2)} t + \text{const}$$

η is fixed by the experimental setup

Free path restriction

Consider two or more spheres

$$\Phi_n = \alpha \sin(\omega t - \Phi_n - \Phi_0) + (n-1)\beta + \nu t)$$



Many particle simulation

Differential equation as seen in the one-particle case.

If two sphere collide:

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} == \begin{pmatrix} 0 & \nu \\ \nu & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

ν coefficient of restitution.

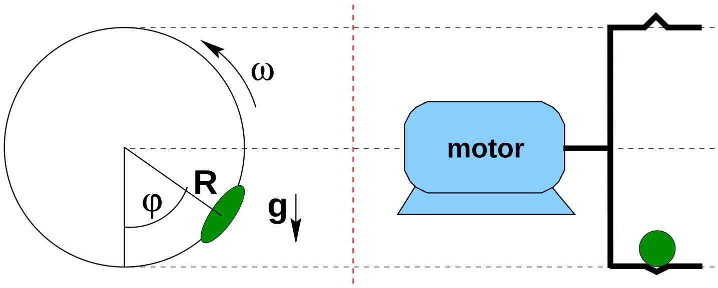
Comparing experiment and theory

ε is fixed by the experimental setup

η is determined from the one-particle experiment

ν is the free parameter

One particle in a rotating drum



- constant angular velocity ω
- sliding, no rolling
- Coefficient of friction μ is a function of the velocity

$$\mathrm{R}\ddot{\varphi}(\mathrm{t}) = -\mathrm{g}\sin\varphi(\mathrm{t}) + \mu(v_{rel}(\mathrm{t})) \cdot \left\{\mathrm{R}\cos\varphi(\mathrm{t}) + \mathrm{R}\dot{\varphi}^2(\mathrm{t})\right\}$$

$$v_{rel} = R \cdot (\omega - \dot{\varphi}) = R \cdot (2\pi f_{\mathrm{motor}} - \dot{\varphi})$$

Non perturbed System

Integrate the system, using $\mu \equiv \mathrm{const}$

$$\dot{\varphi}^2 = -2\frac{\mathrm{g}}{\mathrm{R}}\frac{1}{1+4\mu_0^2} \cdot \left\{(2\mu_0^2 - 1)\cos\varphi - 3\mu_0 \sin\varphi\right\} + 2e^{2\mu_0\varphi} \cdot \mathrm{c}$$

Solve for c and calculate the first derivative

$$\dot{\mathrm{c}} = e^{-2\mu_0\varphi}\dot{\varphi}\underbrace{\left[\ddot{\varphi} + \frac{\mathrm{g}}{\mathrm{R}}\sin\varphi - \mu_0(\mathrm{g}\cos\varphi + \dot{\varphi}^2)\right]}_{\equiv 0}$$

Perturbed System

Perturbation ansatz $\mu(v_{rel}(\mathrm{t}))$:

$$\ddot{\varphi}(\mathrm{t}) = -\frac{\mathrm{g}}{\mathrm{R}}\sin\varphi(\mathrm{t}) + \mu_0 \cdot \left\{\frac{\mathrm{g}}{\mathrm{R}}\cos\varphi(\mathrm{t}) + \dot{\varphi}^2(\mathrm{t})\right\} + \mathrm{perturbation}$$

The velocity dependence of μ is represented by the perturbation.

Averaging Method

We assume that the presence of the perturbation will change the constant of integration c into a slow varying function of time

$$c = c_0(\tau) + \varepsilon c_1(t, \tau) + \cdots$$

with $\tau = \varepsilon t$.

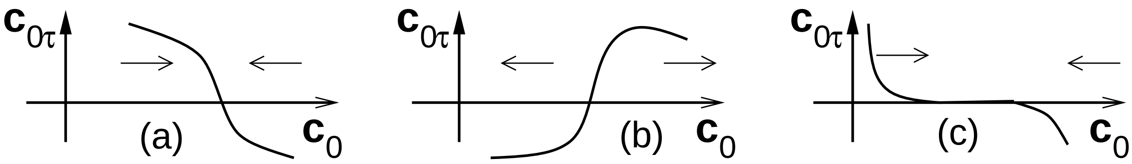
$$\dot{c} = \varepsilon(c_{0\tau}(\tau) + c_{1t}(t, \tau)) + \mathcal{O}(\varepsilon^2)$$

Integrating over one period and using $\dot{\mathrm{c}}$ from the non-perturbed solution, we obtain:

$$\varepsilon c_{0\tau} = -\frac{2}{\mathrm{T}_p} \int_{\varphi^{min}}^{\varphi^{max}} \mathrm{d}\varphi \; \dot{\varphi} e^{-2\mu_0 \varphi} \dot{\varphi} \cdot$$
$$\cdot \varepsilon \left[(\mu(\dot{\varphi}) - \mu_0) \cdot \left(\frac{\mathrm{g}}{\mathrm{R}} \cos\varphi + \dot{\varphi}^2 \right) \right]$$

If $c_{0t}(c_0^*) = 0$, then there is a periodic solution with

$$T(c_0^*) = 2 \int_{\varphi^{min}}^{\varphi^{max}} \mathrm{d}\varphi \; \frac{1}{\dot{\varphi}}$$



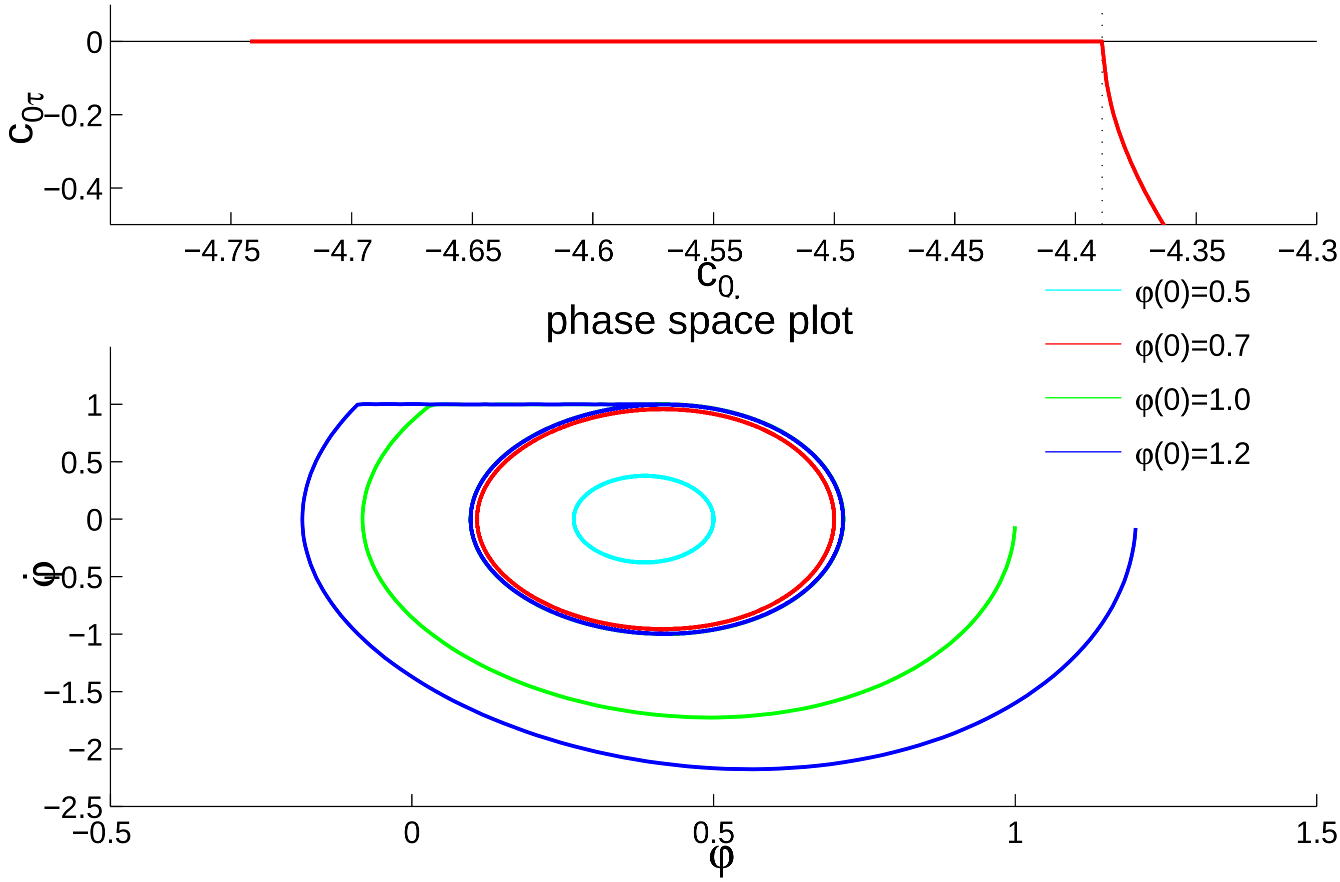
a stable orbit

b unstable orbit

c marginal stable orbit

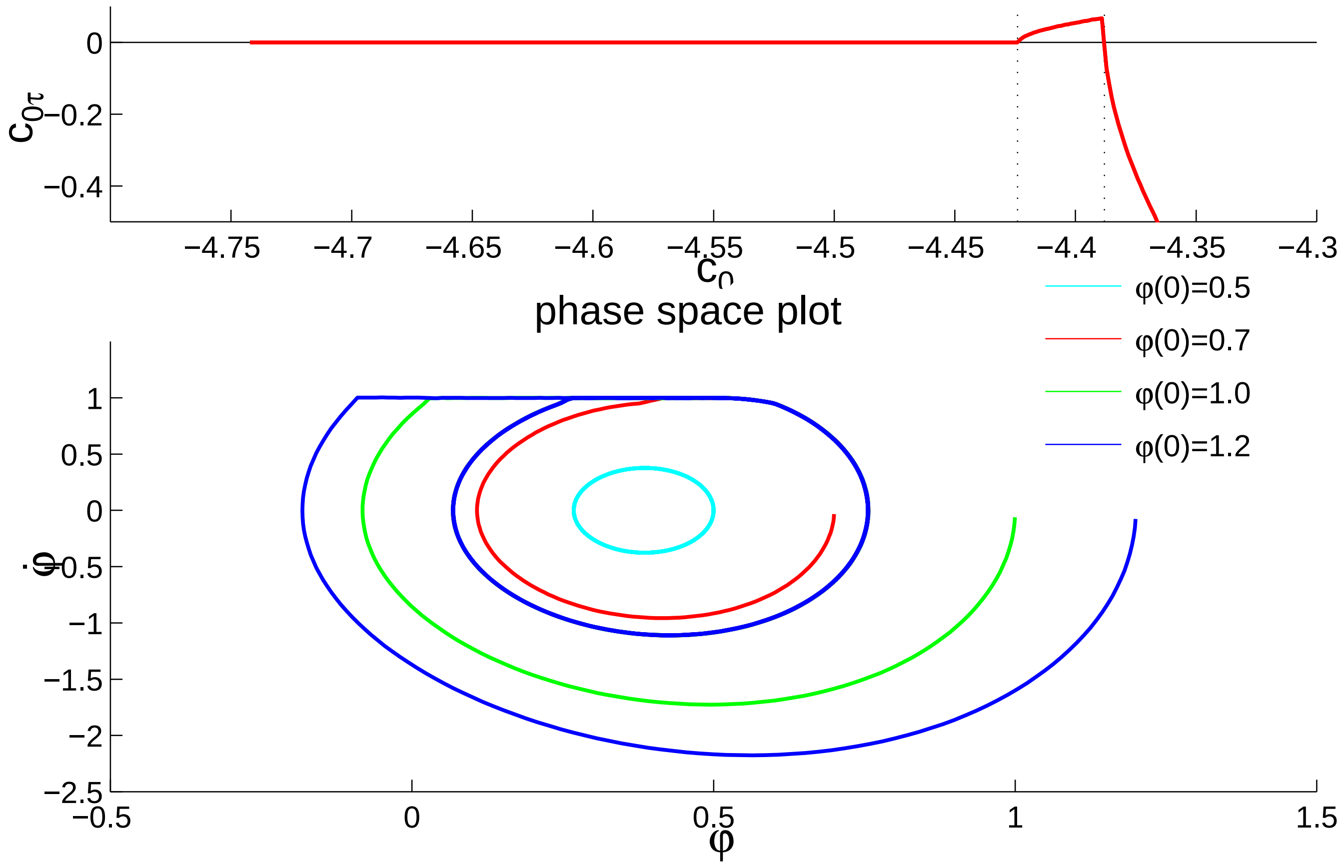
Coulomb

$$\mu(\dot{\varphi}_{rel}) = \begin{cases} \mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} \geq 0 \\ -\mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} < 0 \end{cases}$$



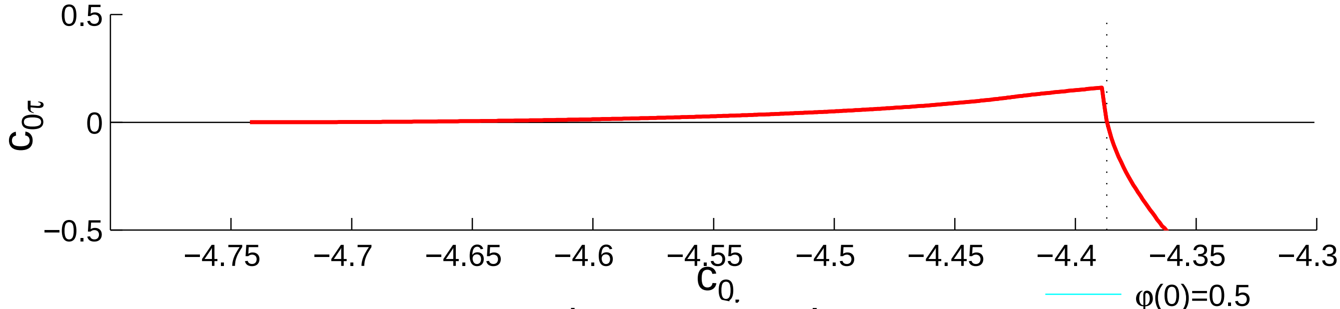
Coulomb + Static Friction

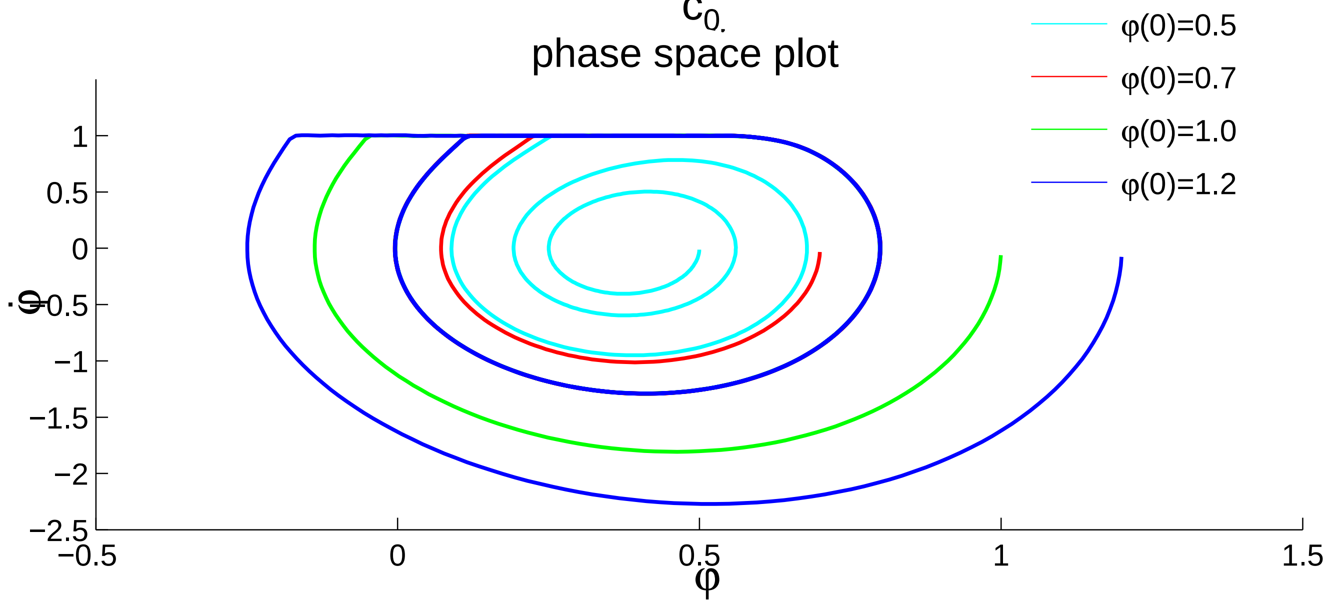
$$\mu(\dot{\varphi}_{rel}) = \begin{cases} \mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} \geq \mathrm{v}_0 \\ \mu_{stat} & \text{if} \quad 0 \leq \dot{\varphi}_{rel} < \mathrm{v}_0 \\ -\mu_{stat} & \text{if} \quad -\mathrm{v}_0 \leq \dot{\varphi}_{rel} < 0 \\ -\mu_{kin} & \text{if} \quad \dot{\varphi}_{rel} < -\mathrm{v}_0 \end{cases} .$$

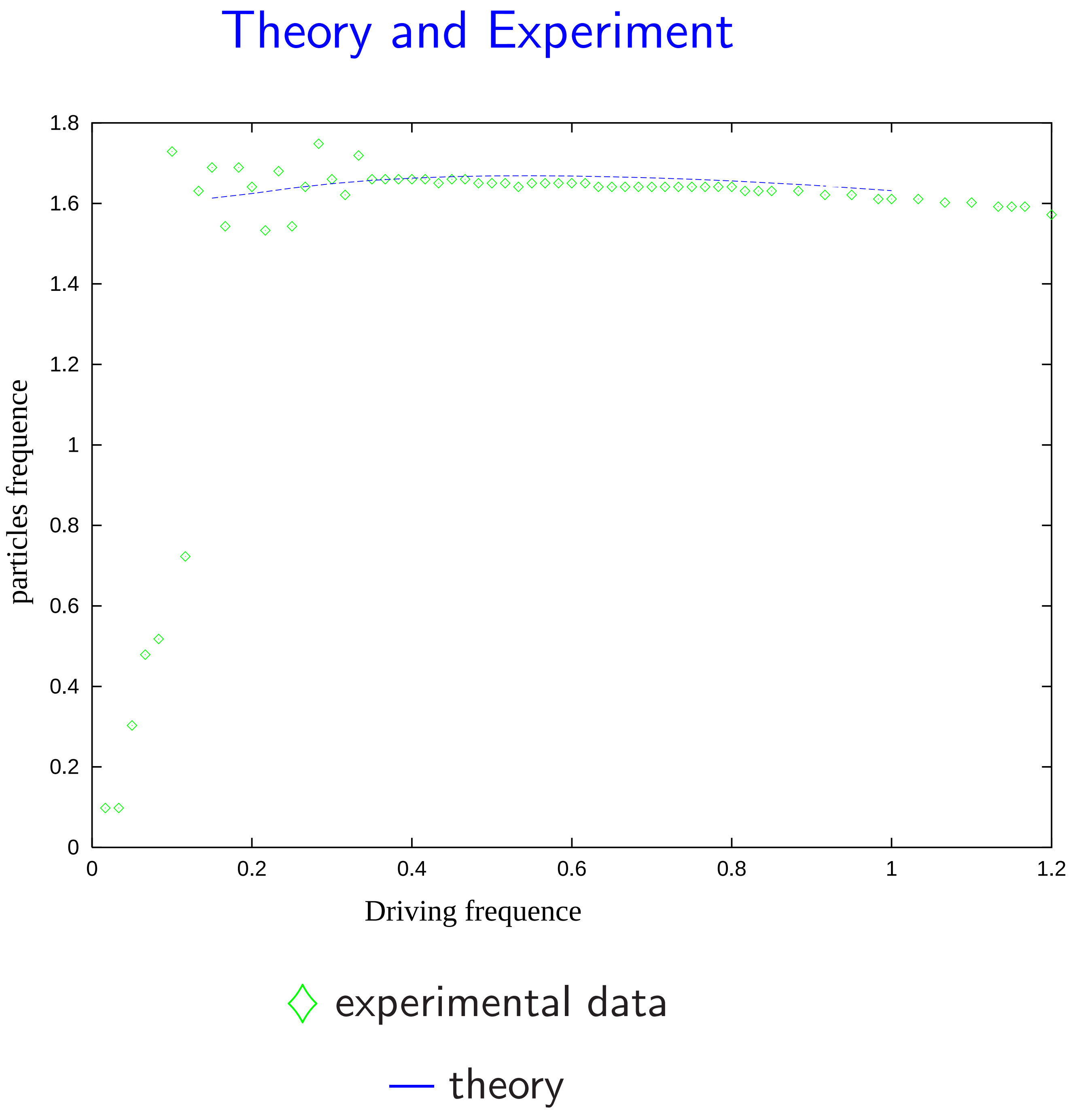


Rabinowicz

$$\mu(\dot\varphi_{rel}) = \begin{cases} \mu_{kin} \cdot |\dot\varphi_{rel}|^{-0.1} & \text{if} \quad \dot\varphi_{rel} \geq \mathrm{v}_0 \\ -\mu_{kin} \cdot |\dot\varphi_{rel}|^{-0.1} & \text{if} \quad \dot\varphi_{rel} < -\mathrm{v}_0 \end{cases}$$






Theory and Experiment
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
particles frequence
Driving frequence
experimental data
theory